Loughton School

Progression in calculations

The national curriculum for mathematics aims to ensure that all pupils:

- Become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- Reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- Can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

This policy outlines the progression for calculations in addition, subtraction, multiplication and division at Loughton School. This policy has been written to ensure consistency and progression throughout the school.

We believe our children will be able to:

· understand important concepts and make connections within mathematics

· show high levels of fluency in performing written and mental calculations

· be taught consistent calculation strategies

· be ready for the next stage of learning

· have a smooth transition between phases

· be able to add, subtract, multiply and divide efficiently

· be competent in fluency, reasoning and problem solving.

Choosing the appropriate strategy and recording mathematically (for calculation in particular) is an important tool both for furthering the understanding of ideas and for communicating those ideas to others.

A useful written method is one that helps children carry out a calculation and can be understood by others. Written methods are complementary to mental methods and should not be seen as separate from them.

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. It is important children acquire secure mental methods of calculation and one efficient written method of calculation for addition, subtraction, multiplication and division which they know they can rely on when mental methods are not appropriate.

Mathematical vocabulary

High expectations of the mathematical language used are essential, with staff only accepting what is correct. Consistency across the school is key.

Correct terminology (since new 2014 curriculum)	**Incorrect terminology (what you may have used previously)**
ones	units
is equal to (is the same as)	equals
zero	oh (the letter o)
exchange / exchanging	stealing
regrouping	borrowing
calculation / equation / number sentence	carrying
commutative	generic term of ‘sum’
bridge	swap/move

whole and part – particularly useful when using bar modelling

Mathematical representation

Consistency across the school for number representation: Headings for columns need to be consistent so that children understand the value of each digit.

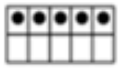
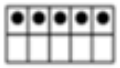
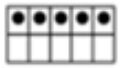
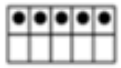
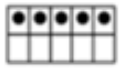
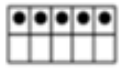
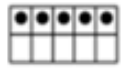
M Hth Tth Th H T O . $\frac{1}{10}$ $\frac{1}{100}$ $\frac{1}{1000}$

Bar modelling should be used as a regular representation throughout topics.

part + part = whole
part + part = whole
whole – part = part
whole – part = part

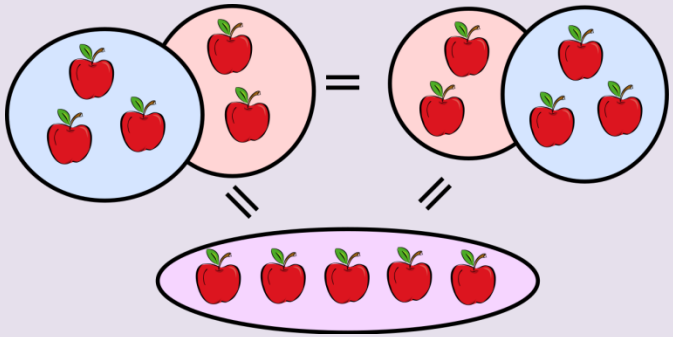
Adapt for multiplication, division, fractions etc

Progression in Manipulatives

Foundation	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Real life objects	Real life objects	Real life objects	Real life objects	Real life objects	Real life objects	Real life objects
0 - 9 digit cards	0 - 9 digit cards	0 - 9 digit cards	0 - 9 digit cards	0 - 9 digit cards	0 - 9 digit cards	0 - 9 digit cards
Number track to 10	Number line to 20	Number line to 100	Number line to 100	Number line including negative numbers	Number line including negative numbers	Number line including negative numbers
Counting stick	Counting stick	Counting stick	Counting stick	Counting stick	Counting stick	Counting stick
Tens frame 	Tens frame 	Tens frame 	Tens frame 	Tens frame 	Tens frame 	Tens frame 
Bead strings 10	Bead strings 20	Bead strings 100	Bead strings 100	Bead strings 100	Bead strings 100	Bead strings 100
Interlocking cubes - use one colour to represent one amount	Interlocking cubes - use one colour to represent one amount	Base 10	Base 10	Base 10	Base 10	Base 10
		Place Value counters	Place Value counters	Place Value counters	Place Value counters	Place Value counters
	Arrow cards - tens and ones	Arrow card - tens and ones	Arrow card - Hundreds, tens and ones	Arrow card - thousands, hundreds, tens and ones	Arrow card - thousands, hundreds, tens and ones, including decimals	Arrow card - thousands, hundreds, tens and ones, including decimals
Part part whole model	Part part whole model	Part part whole model	Part part whole model	Part part whole model	Part part whole model	Part part whole model
Bar model with real life model objects	Bar model with real life objects, pictorial representations eg counters	Bar models with counters/ base 10 progressing to numbers	Bar models with numbers	Bar models with numbers	Bar models with numbers	Bar models with numbers
Numicon	Numicon	Numicon	Numicon	Numicon	Numicon	Numicon
			Cuisenaire rods	Cuisenaire rods	Cuisenaire rods	Cuisenaire rods
Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters
Multilink - use one colour to model an amount	Multilink - use one colour to model an amount	Multilink - use one colour to model an amount	Multilink - use one colour to model an amount	Multilink - use one colour to model an amount	Multilink - use one colour to model an amount	Multilink - use one colour to model an amount

Progression in calculations

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on. Regrouping to make 10.	Adding three single digits. Column method (tens and ones) - no exchanging.	Column method with exchange (up to 3 digits)	Column method with exchange (up to 4 digits)	Column method with exchange (with more than 4 digits) Column method with decimals - with the same amount of decimal places.	Column method with exchange (with more than 4 digits) Column method with decimals - with a different amount of decimal places.
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10	Counting back Find the difference Part whole model Make 10 Column method - no exchanging	Column method with exchange (up to 3 digits)	Column method with exchange (up to 4 digits)	Column method with exchange (with more than 4 digits) Column method with decimals - with the same amount of decimal places.	Column method with exchange (with more than 4 digits) Column method with decimals - with a different amount of decimal places.
Multiplication	Doubling Counting in multiples Arrays	Doubling Counting in multiples Arrays- showing commutative multiplication Repeated addition	Counting in multiples Arrays- showing commutative multiplication Repeated addition Visual grid method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit multiplied by 1 or 2 digit)	Column multiplication (multi-digit numbers up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping	Division as grouping Division with arrays	Division with arrays Division with a remainder Short division (2 digit by a 1 digit, pictorial and abstract representation)	Division with arrays Division with a remainder Short division (3 digit by a 1 digit, pictorial and abstract representation)	Short division (up to 4 digits by 1 digit number interpret remainders appropriately for the context)	Short division Long division (up to a 4 digit number by a 2 digit number - interpret remainders whole numbers, fractions or a round)



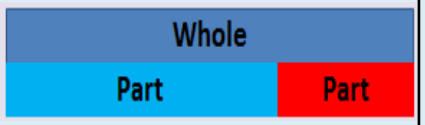
Addition

Key Vocabulary:

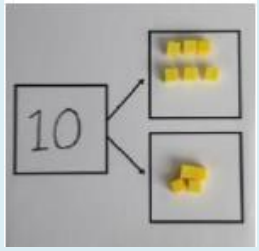
add, addition, plus, and, count on, more, sum, total, altogether, increase, balance, much, inverse, double, near double.

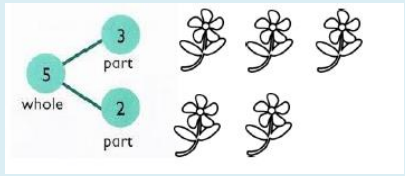
Teachers need to model the language of addend + addend = sum/total.

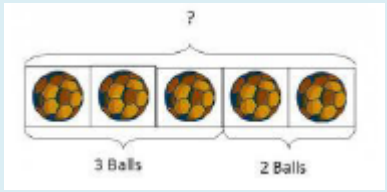
Objective and strategies	Concrete	Pictorial	Abstract
Combining two parts to make a part-part-whole model. (Aggregation)	Use cubes to add two numbers together as a group or in a bar.	Use pictures to add two numbers together as a group or in a bar.	Use the part-part-whole diagram as shown to move into the abstract.

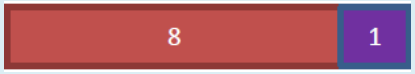


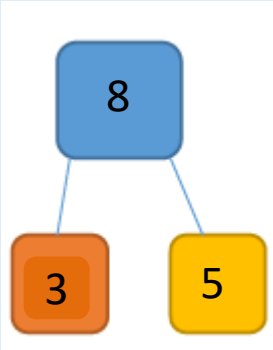
part + part = whole
part + part = whole









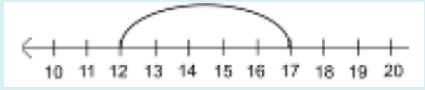


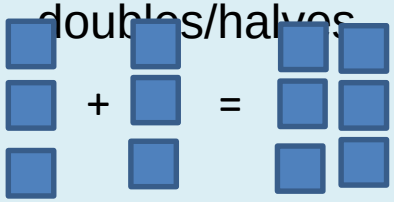
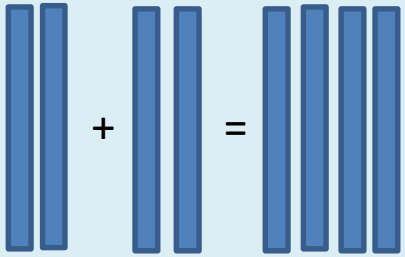
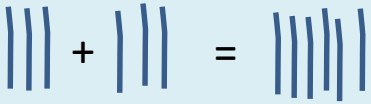
4 + 3 = 7

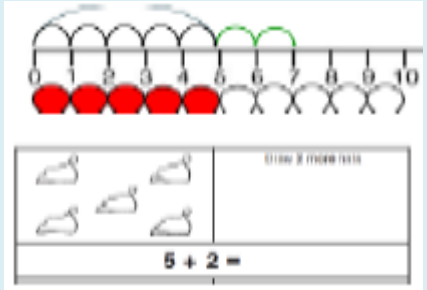
10= 6 + 4

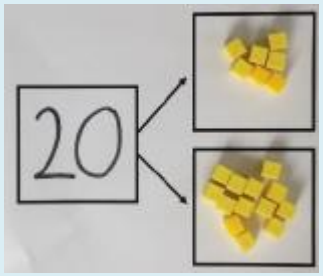
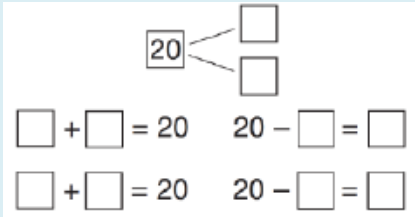
___ is a part and ___ is a part, the parts add together to make the whole.

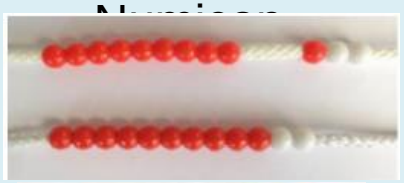
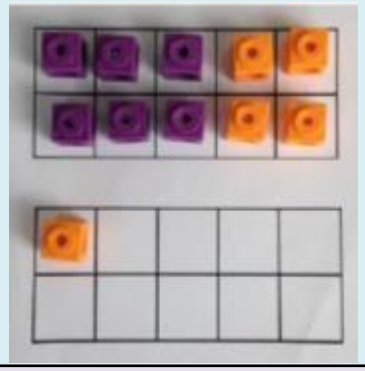
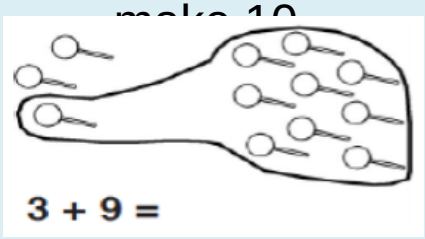
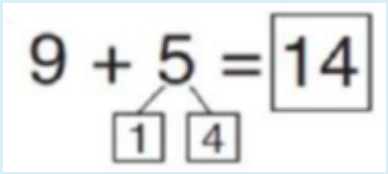
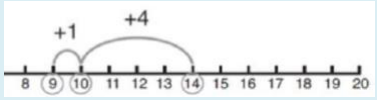
The whole is __.

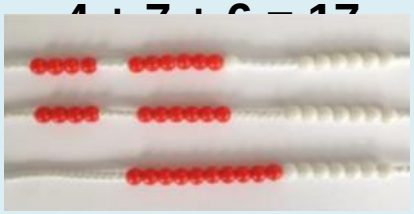
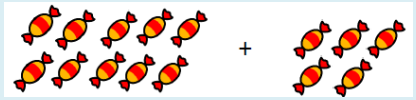
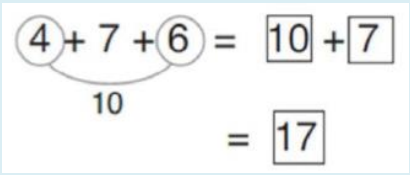
Objective and strategies	Concrete	Pictorial	Abstract
Starting at the bigger number and counting on. (Augmentation) Step 1: The greater number is _____. Step 2: The smaller number is _____. Step 3: I start with _____ I count on _____. The total is ___	Start with the greater number on the bead string and count on the smaller number one by one to find the sum/total.  Use a variety of resources to make the bigger number and then count on in ones while adding on the smaller number.	Start at the greater number on the number line and count on the smaller number in ones, or in one jump, to find the sum/ total.  	Place the greater number in your head and count on the smaller number to find the sum/total. **5 + 12 = 17** *image purposefully shown as children will see calculations in a range of ways and they need to use the commutative law and still begin with the greatest number.

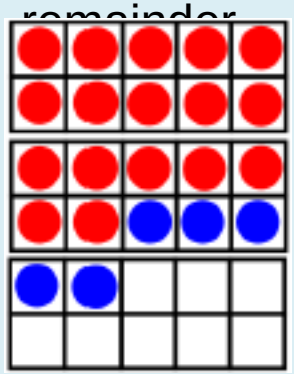
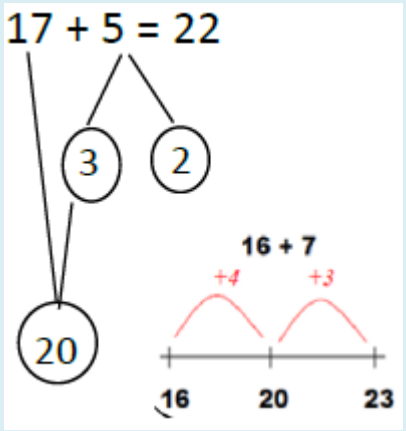
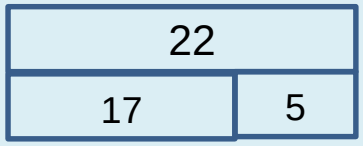
Objective and strategies	Concrete	Pictorial	Abstract
Using known facts. Adding multiples of 10. I know that __ + __ = __ Then I also know __ tens + __ tens = __ tens Extend to hundreds, tenths etc.	Use cubes, objects, base 10, place value counters and Numicon to make number bonds, doubles/halves   	Draw representations of H, T and O using base 10 equipment or equivalent.   	3 + 7 = 10 30 + 70 = 100 300 + 700 = 1000 3 tens + 5 tens = 8 tens 30 + 50 = 80

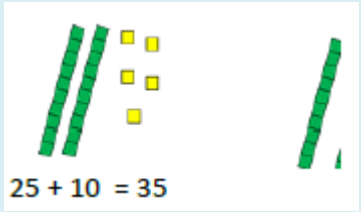
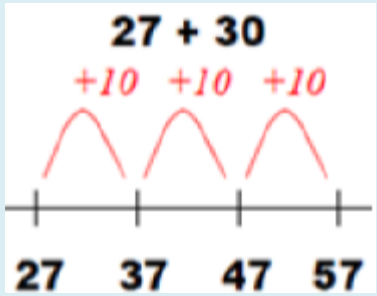
Objective and strategies	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20. I know that ____ (use the language in the abstract section)	Use bead strings, cubes, objects, base 10, place value counters and Numicon to show 2 more than 5. Show if 2 more than 5 is 7, then 2 less than 7 is 5. 	Representations of number lines, drawing of objects to show number bonds and related subtraction facts. 	Emphasis should be on the language: “One more than 5 is equal to 6” “2 more than 5 is 7” Eight is three more than five”

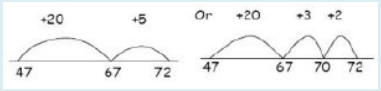
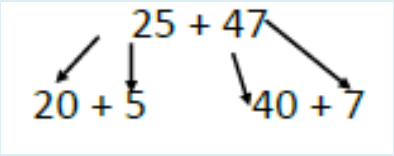
Objective and strategies	Concrete	Pictorial	Abstract
Use known number facts to find the inverse. Solve missing number problems. If I know that part + part = whole Then I also know that whole – part = part 	Children explore way of making numbers within 20. Use part-part-whole diagrams, tens frames and Numicon.  Progress to increasingly larger numbers.	Representation of part-part-whole diagram, tens frame, bar model and Numicon.  Progress to increasingly larger numbers.	Make fact families using known facts. □ + 1 = 16 1 + □ = 16 16 - □ = 1 16 - 1 = □ Progress to increasingly larger numbers.

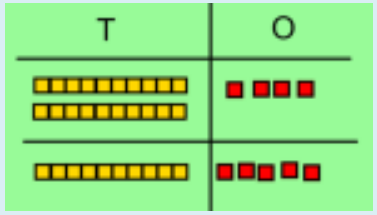
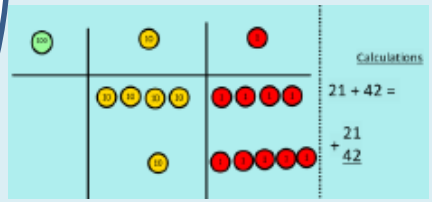
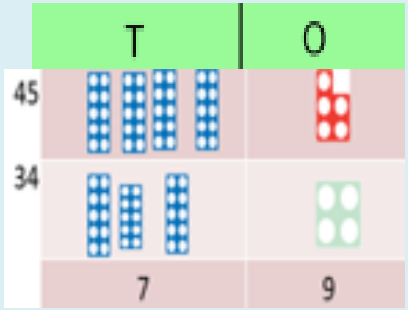
Objective and strategies	Concrete	Pictorial	Abstract
Regrouping to make 10. 7 + 4 = 11 Step 1: 7 + __ = 10 Step 2: 3 + __ = 4 Step 3: 10 + 1 = __	Start with the bigger number and use the smaller number to make 10, Use tens frame, bead strings and  	Use pictures or a number line. Partition and regroup the smaller number to  3 + 9 =  9 + 5 = 14 1 4  +1 +4 8 9 10 11 12 13 14 15 16 17 18 19 20	If I am at 7, how many more do I need to make 10. How many more do I add on now? 7 + 4 = 11 7 + 3 + 1 = 11 10 + 1 = 11

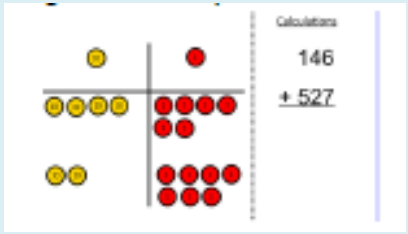
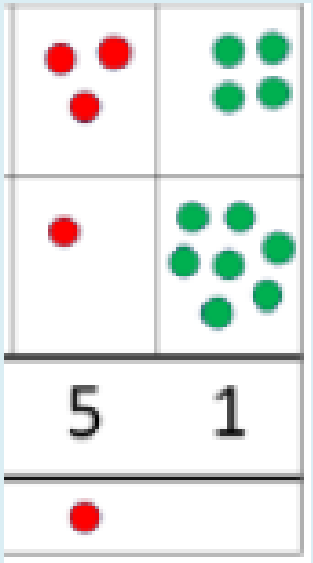
Objective and strategies	Concrete	Pictorial	Abstract
Adding three single digits. Step 1: look for a known fact (doubles, number bond etc) Step 2: calculate known fact. Step 3: add on the remaining number to find the sum/total.	Add together three groups of objects. Put two of the groups together to make 10 and then add on the third group. Put 4 and 6 together to make 10 and then add on 7. 	Representations of combining numbers to make 10. Draw pictures to show combining groups to make 10.   15 3 | 7 | 5	Combine the two numbers to make 10 and add on the remaining number. 

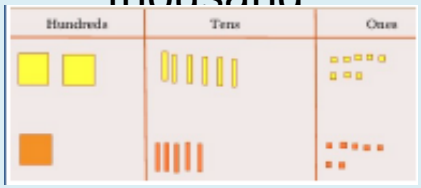
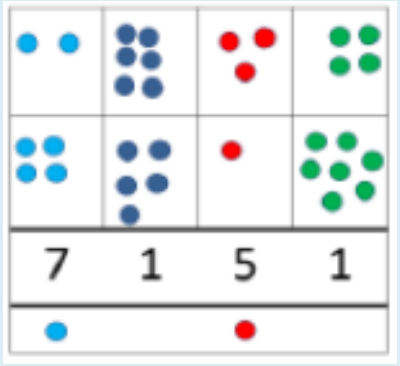
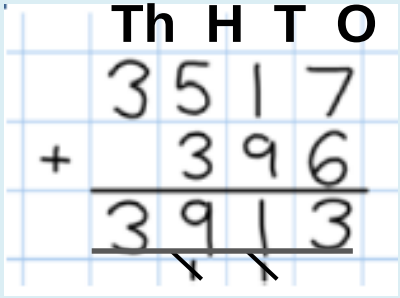
Objective and strategies	Concrete	Pictorial	Abstract
Add a 2 digit number and ones. Stem sentences below: Step 1: find the greatest number. Step 2: find how many more to the next whole ten. Step 3 : 17 + __ = next whole ten Step 4: 3 + __ = 5 Step 5: 20 + 2 =	Use tens frame, base 10 and Numicon to make a whole ten, then add on the remainder.  Children to explore patterns in calculations which 17 + 5 = 22 27 + 5 = 32 37 + 5 = 42	Use part-part-whole and number lines to model. 	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22 – 5 = 17 22 – 17 = 5 22 = 5 + 17  Extend to missing number calculations □ = □ + 5

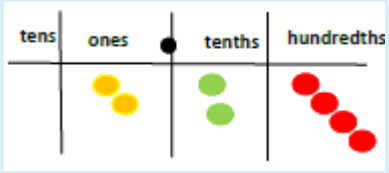
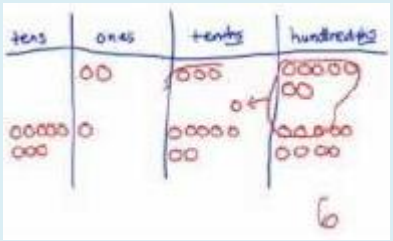
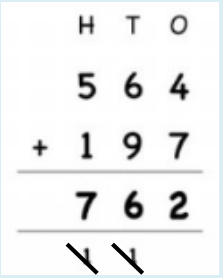
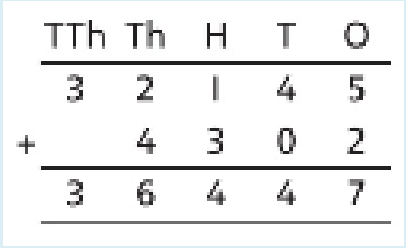
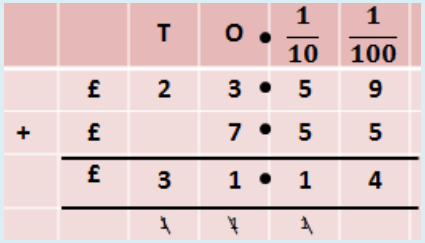
Objective and strategies	Concrete	Pictorial	Abstract
Add a 2 digit number and tens. __ is __ tens and __ ones. __ tens and __ ones + __ tens = __ tens and __ ones. ____ + ____ is equal to __.	Using base 10 equipment, arrow cards and Numicon, explore that the ones digit does not change.  	Use pictures or number lines to show adding multiples of 10.  	Explore patterns in calculations including missing numbers and calculations that begin with the 27 + 10 = 37 27 + 20 = 47 27 + ☐ = 57 57 = 27 + ☐

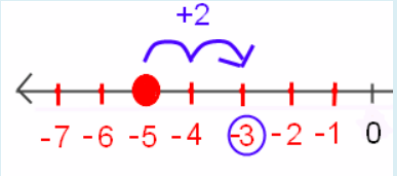
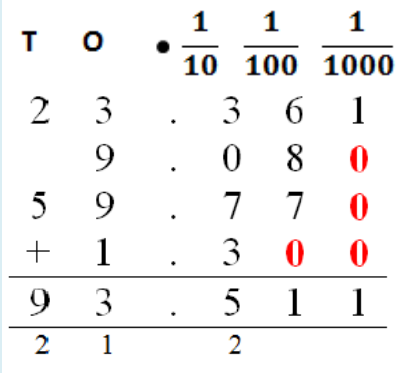
Objective and strategies	Concrete	Pictorial	Abstract
Add two 2-digit numbers. Stem sentences below: 25 is 2 tens and 5 ones. 47 is 4 tens and 7 ones. 2 tens + 4 tens = 6 tens 5 ones + 7 ones = 12 ones 12 ones = 1 ten and 2 ones 6 tens + 1 ten = 7 tens 7 tens + 2 ones = 72	Model using base 10, place value counters and Numicon. 	Use number lines to bridge 10. Use part-part-whole diagram if necessary. 47 + 25 shown as 47 + 20 + 3 + 2  	 20 + 40 = 60 5 + 7 = 12 60 + 12 = 72

Objective and strategies	Concrete	Pictorial	Abstract
Column addition – no regrouping. Add numbers with up to 3 digits. Step 1: add the ones *teacher language - begin with the least significant digit* Step 2: add the tens Step 3: add the hundreds	Model using base 10 and Numicon, then place value counters. Add together starting with the smallest value digit.   	Draw place value counters using a tens and ones frame.   	Formal column addition without regrouping. Adding the smallest value digit first. Add the ones first, then the tens, then the hundreds H T O 2 2 3 + 1 1 4 3 3 7

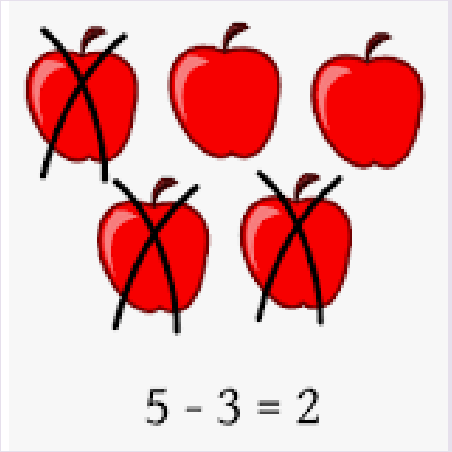
Objective and strategies	Concrete	Pictorial	Abstract
Column addition with regrouping. Step 1: 7 ones + 4 ones Step 2: 6 tens + 2 tens = 8 tens Step 3: add to find the sum/total	Exchange 10 ones for 1 ten. Model using Numicon and place value counters, using a HTO frame. 	Draw a representation of the grid to further support understanding, regrouping underneath the line. 	Formal column addition with exchange. Begin with expanded method and progress to concise. **T O** 6 7 + 2 4 1 1 (7 + 4) + 8 0 (60 + 20) 9 1 2 3 . 3 6 1 9 . 0 8 0 5 9 . 7 7 0 + 1 . 3 0 0 9 3 . 5 1 1 2 1 2

Objective and strategies	Concrete	Pictorial	Abstract
Y4 **Add numbers with up to four digits.** Stem sentences below: **Step 1**: add the ones (exchange if necessary) **Step 2**: add the tens (exchange if necessary) **Step 3**: add the hundreds (exchange if necessary) **Step 4**: add the thousands (exchange if necessary)	Children to continue to use base 10 or place value counters to add, exchanging 10 ones for 1 ten, 10 tens for 1 hundred and 10 hundreds for 1 thousand 	Draw representations using a place value grid. 	Continue previous work to exchange and regroup hundreds as well as tens. Relate to measures and money. 

Objective and strategies	Concrete	Pictorial	Abstract
Y5 Add numbers with more than 4 digits. Add decimals with 2 decimal places including money. Stem sentences as for Y4	As year 4. Regular use of place value counters and model exchange and regrouping for addition. 	As year 4. Draw representations using a place value grid. 2.37 + 81.79 	Formal column addition with exchange and regrouping.   

Objective and strategies	Concrete	Pictorial	Abstract
Y6 Add several numbers of increasing complexity. Include adding money, measures and decimals with different numbers of decimal places. Stem sentences as for Y4/Y5	As year 5	As year 5 Use number lines to introduce adding negative numbers. 	Formal column addition. Use place holders for decimals with different numbers of decimal places. 

T	O	•	1/10	1/100	1/1000
2	3	.	3	6	1
	9	.	0	8	0
5	9	.	7	7	0
+	1	.	3	0	0
9	3	.	5	1	1
2	1		2		

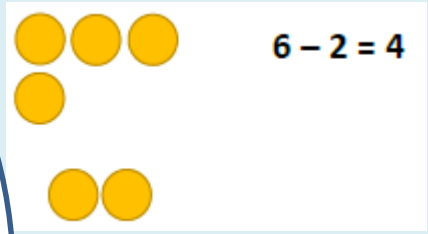
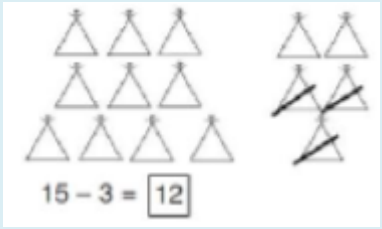


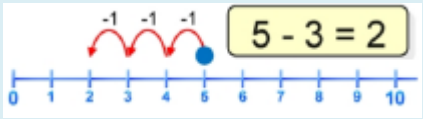
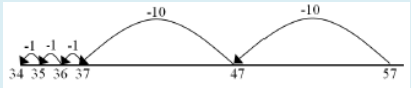
Subtraction

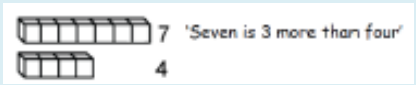
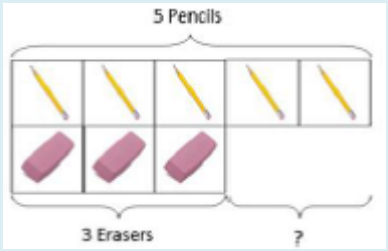
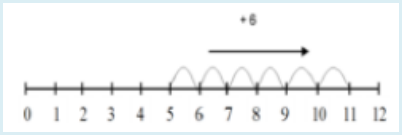
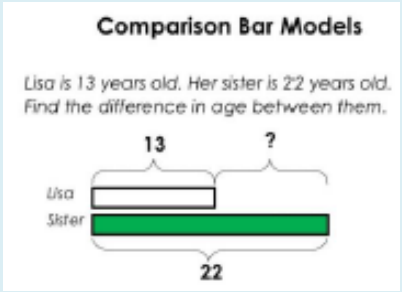
Key Vocabulary:

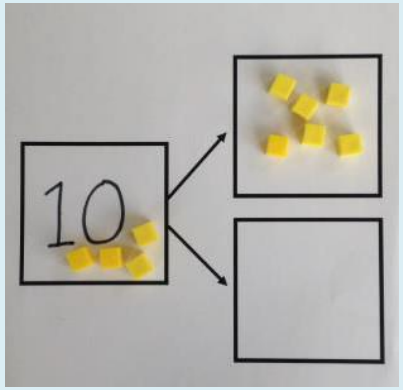
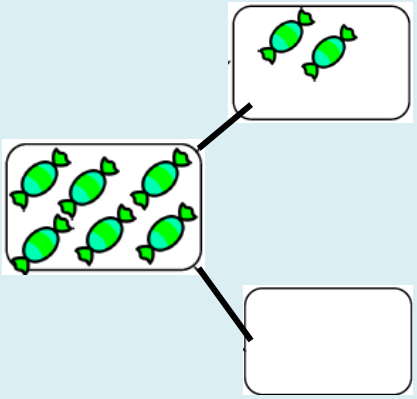
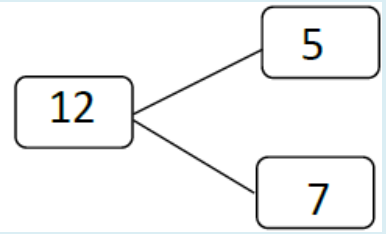
Subtract, subtraction, minus, less, take away, decrease, fewer, difference, exchange.

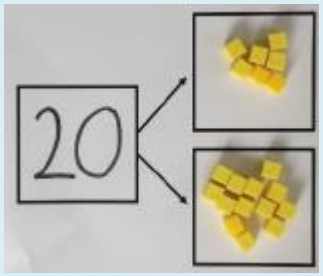
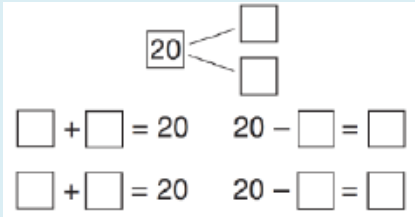
Teachers need to model the language of minuend - subtrahend = difference.

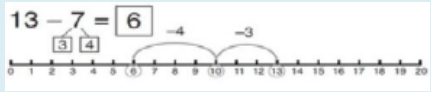
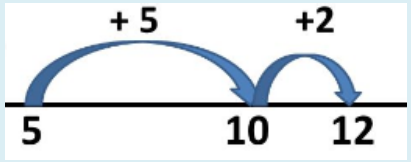
Objective and strategies	Concrete	Pictorial	Abstract
Taking away ones Step 1: The greater number is _____. Step 2: The smaller number is _____. Step 3: I start with _____ I count back ___.	Use physical objects, two tone counters, cubes etc to show how objects can be taken away. 	Cross out drawn objects to show what has been taken away.  	Written number sentence. Solve using fingers or using known facts. 8 – 2 = 6

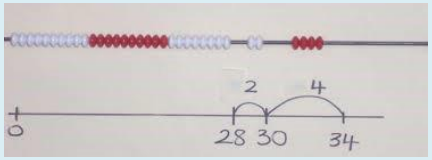
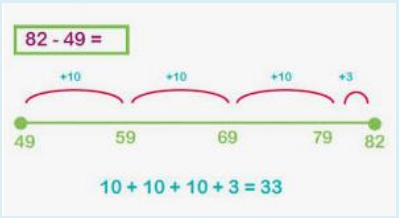
Objective and strategies	Concrete	Pictorial	Abstract
Counting back Step 1: The greater number is _____. Step 2: The smaller number is _____. Step 3: I start with _____ I count back ___.	Move the beads along the bead string as you count backwards. **13 – 4**  Move objects away from the group, counting backwards. **7 – 2** 	Count back in ones on a number line.  Start at the bigger number and count back the smaller number showing the jumps on the number line.  	Put 13 in your head, count back 4. What number are you at?

Objective and strategies	Concrete	Pictorial	Abstract
Find the difference Start with the subtrahend. Count on to the minuend. The difference is ____.	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference.  Use basic bar models with items to find the difference.	Count on to find the difference.  Draw bars to find the difference between 2 numbers.  	Hannah has 23 sandwiches, Helen has 15 sandwiches. How many more does Hannah have than Helen? Write a number sentence to find the difference between the number of sandwiches.

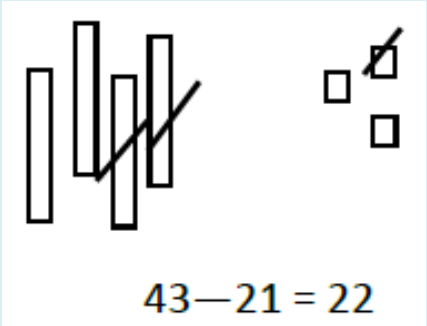
Objective and strategies	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20. I know that ____ (use the language in the abstract section)	Link to addition. Use Part Part Whole to model the inverse.  If 10 is the whole and 6 is one of the parts, what is the other part? 10—6 = 4	Use pictorial representations to show the whole and the parts. 	Move to using numbers within the part whole model.  Make fact families using known facts. Progress to increasingly larger numbers. Emphasis should be on the language: “One less than 5 is equal to 4” “Eight is three

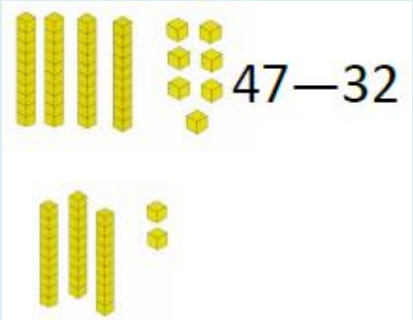
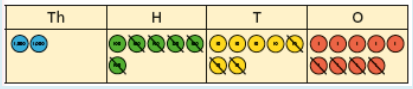
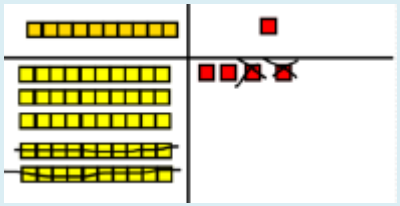
Objective and strategies	Concrete	Pictorial	Abstract
Use known number facts to find the inverse. Solve missing number problems. If I know that part + part = whole Then I also know that whole – part = part 	Children explore ways of making numbers within 20. Use part-part-whole diagrams, tens frames and Numicon.  Progress to increasingly larger numbers.	Representation of part-part-whole diagram, tens frame, bar model and Numicon.  Progress to increasingly larger numbers.	Make fact families using known facts. □ + 1 = 16 1 + □ = 16 16 - □ = 1 16 - 1 = □ Progress to increasingly larger numbers.

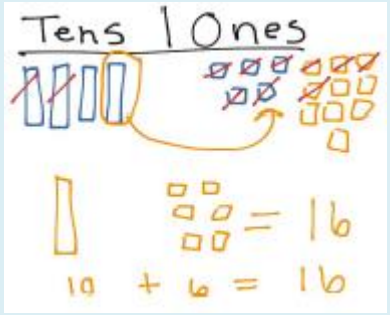
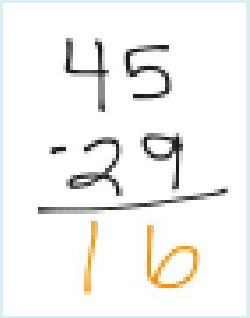
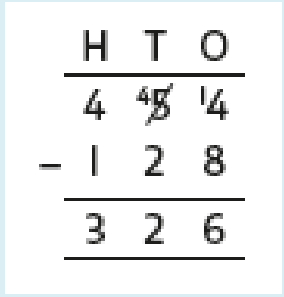
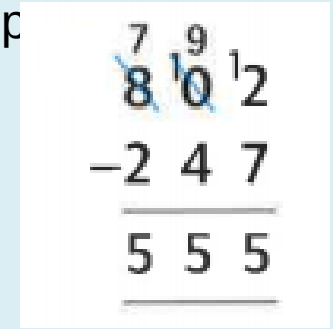
Objective and strategies	Concrete	Pictorial	Abstract
Bridge 10 13 – 7 = 6 Step 1: 13 - __ = 10 Step 2: 3 + __ = 7 Step 3: 10 - 4 = __	Use a tens frame and counters or double sided counters to bridge 10. 14 – 5 = 9  Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have subtracted 5.	Counting back: 13—7  Subtract 3 first, then another 4. Use ten as the stopping point. Counting on: 12 – 5 = 7  	16 - 8 How many do we take off first to get to 10? How many left to take off?

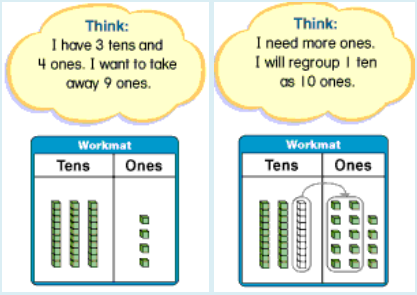
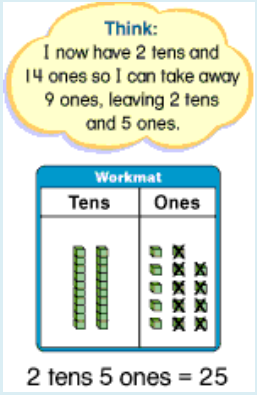
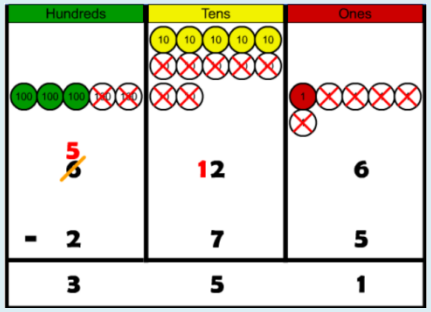
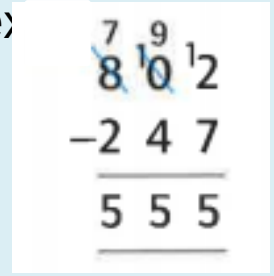
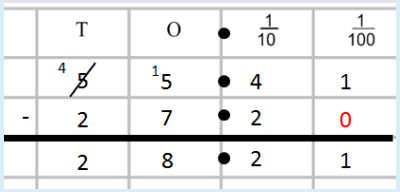
Objective and strategies	Concrete	Pictorial	Abstract
Make ten strategy *Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.*	Use a bead string to model counting to next ten and then counting on the rest. 34—28 	Use a number line to count on to next ten and then the rest. 	93—76 = 17 Mentally: begin at 76. Add on 4 to 80, then 10 more to 90, then add 3 to reach 93. 4 + 10 + 3 = 17

Stem sentence in the notes.

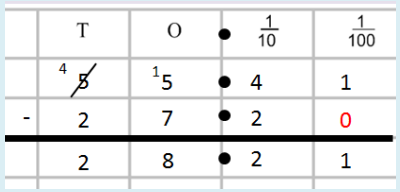
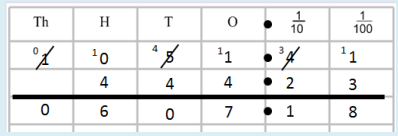
Objective and strategies	Concrete	Pictorial	Abstract
Partitioning to subtract without re-grouping. STEM sentence in the notes.	34—13 = 21 Use base 10 to show how to partition the number when subtracting without re 	Children draw representations of base 10 and cross off. 	43—21 = 22 Mentally subtract the tens and subtract the ones.

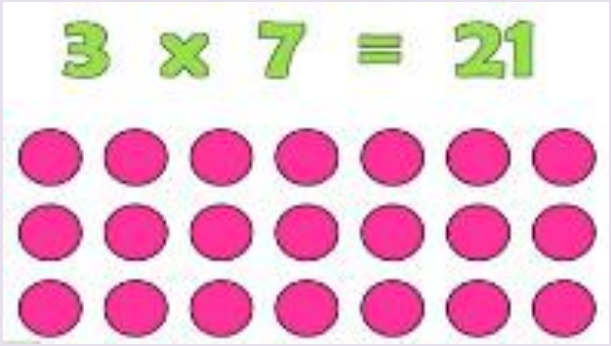
Objective and strategies	Concrete	Pictorial	Abstract
Column subtraction without regrouping numbers up to 3 digits. Step 1: subtract the ones *teacher language - begin with the least significant digit* Step 2: subtract the tens Step 3: subtract the hundreds	Use base 10 or place value counters to model.  	Draw representations to support understanding. 	Formal column subtraction without exchange. 2679 – 534 = ☐ Th H T O 2 6 7 9 – 5 3 4 2 1 4 5

Objective and strategies	Concrete	Pictorial	Abstract
Column subtraction with regrouping up to 3 digits Stem sentences below: **Step 1**: subtract the ones (exchange if necessary) **Step 2**: subtract the tens (exchange if necessary) **Step 3**: subtract the hundreds (exchange if necessary) **Step 4**: subtract the thousands (exchange if necessary)	Begin with base 10 or Numicon, moving on to place value counters. Model the exchange of one ten for ten ones. Use the phrase 'exchange one ten for ten ones' 	Children may draw base ten or PV counters and cross off.  	Formal column subtraction with exchange.  Explicitly teach the following p 

Objective and strategies	Concrete	Pictorial	Abstract
Year 4 - Subtract up to 4 digits with regrouping. *Introduce decimal subtraction through context of money and measure.* Stem sentences as for previous slide	Model process of exchange using base ten and then move to PV counters.  	Children to draw place value counters and show their exchange. 	Column subtraction with ex  Use place value to line up decimal points and use a place holder to show value of empty places 

Objective and strategies	Concrete	Pictorial	Abstract
Year 5 - Subtract with at least 4 digits, including money and measures. *Subtract with decimal values, including mixtures of integers and decimals, correctly aligning the decimal point.* Stem sentences as for Y4	As Year 4	As Year 4	Column subtraction. TTh Th H T O $^{7}\not{8}$ $^{1}2$ $^{6}\not{7}$ $^{1}0$ 6 − 3 9 4 1 5 4 3 2 9 1 Use place value to line up decimal points and use a place holder to show value of empty places. T O • $\frac{1}{10}$ $\frac{1}{100}$ $^{4}\not{5}$ $^{1}5$ • 4 1 - 2 7 • 2 0 2 8 • 2 1

Objective and strategies	Concrete	Pictorial	Abstract
Year 6 Subtract with increasingly large and more complex numbers and decimal values. Stem sentences as for Y4/Y5	As Year 4/5	As Year 4/5	Column subtraction. Use place value to line up decimal points and use a place holder to show value of empty places.  

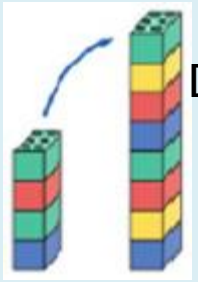
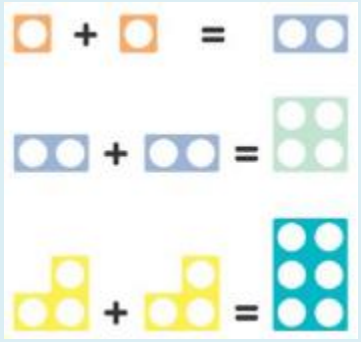
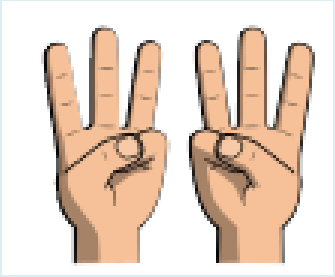


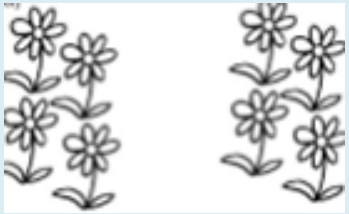
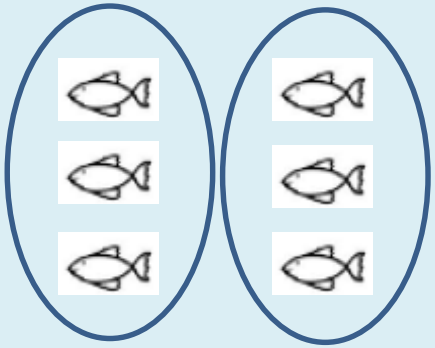
Multiplication

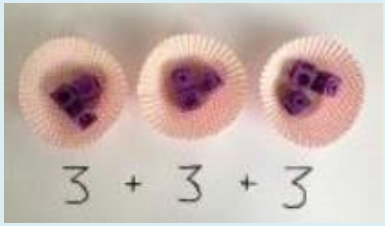
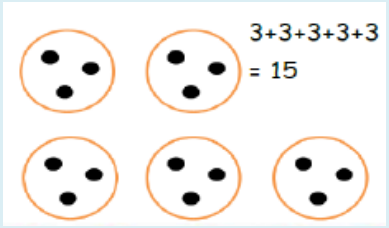
Key Vocabulary:

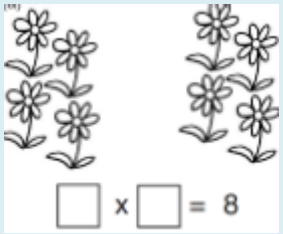
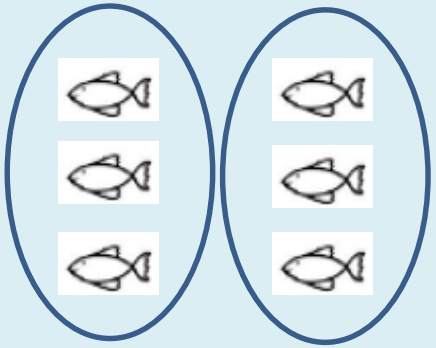
Multiply, times, equal groups, double, commutative, array, row, column, multiple, factor, product, common factor, common multiple, prime, composite, squared (x^2), cubed (x^3), order of operations, brackets, inverse operation

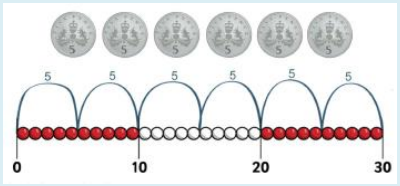
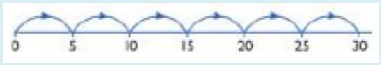
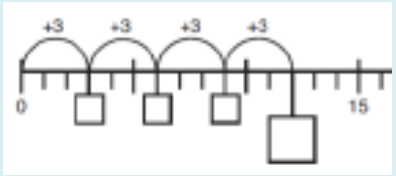
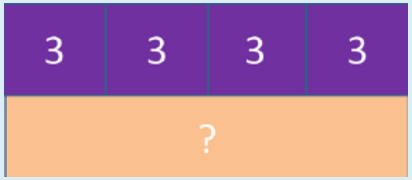
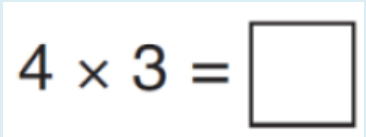
Teachers need to model the language of factor x factor = product.

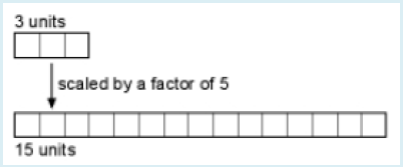
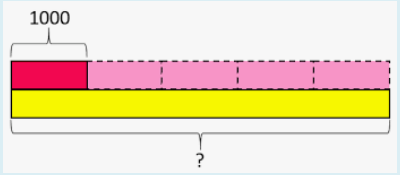
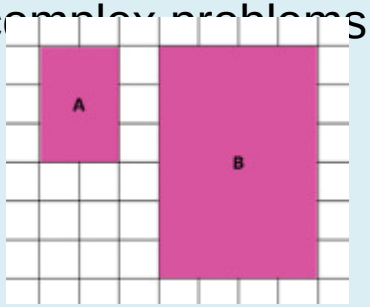
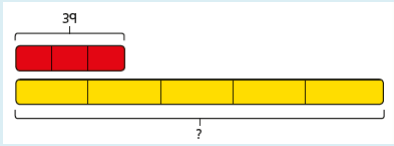
Objective and strategies	Concrete	Pictorial	Abstract
Doubling numbers to 10. Double ___ is equal to ___	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling.  Double 4 is 8 	Draw pictures to show how to double numbers . Double 4 is 8 	Rolling numbers, step counting and using fingers to double numbers to 10.  Double 3 is equal to 6

Objective and strategies	Concrete	Pictorial	Abstract
Making equal groups. There are __ groups of 3. There are __ groups, __ in each group.	Use manipulatives to create equal groups.   2 equal groups of 4	Draw  to show 2 equal groups of 3 	Language to describe the number sentence using “equal groups” and “total” “2 equal groups of 4 have a total of 8”

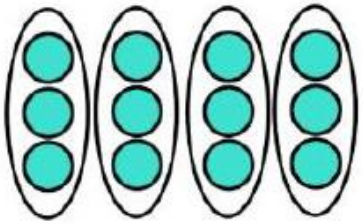
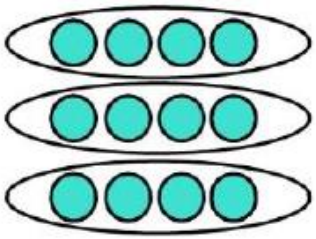
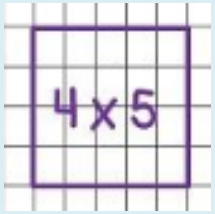
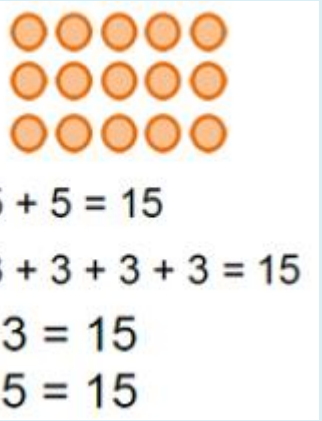
Objective and strategies	Concrete	Pictorial	Abstract
Repeated addition. 4 groups of 3 = __+__+__ +__	Use different objects to add equal groups.   	Use pictorial representations including number lines to solve problems .   	Write addition sentences to describe objects and pictures. 

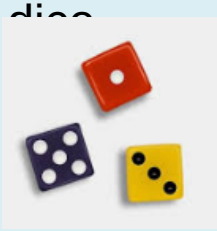
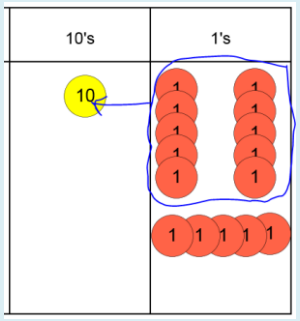
Objective and strategies	Concrete	Pictorial	Abstract
Counting in equal groups to find the product. There are __ groups of 3. There are __ groups, __ in each group, so __ altogether. __ groups of __ is equal to __	Use manipulatives to create equal groups.  	Draw  to show 2 x 3 = 6 	Rolling numbers, step counting and known facts. Language to describe the number sentence using “groups” and “total” 2 x 4 = 8 “2 groups of 4 have a total of 8”

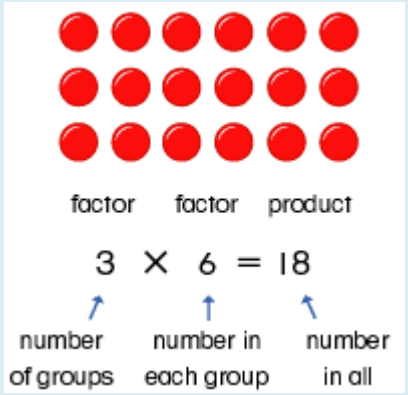
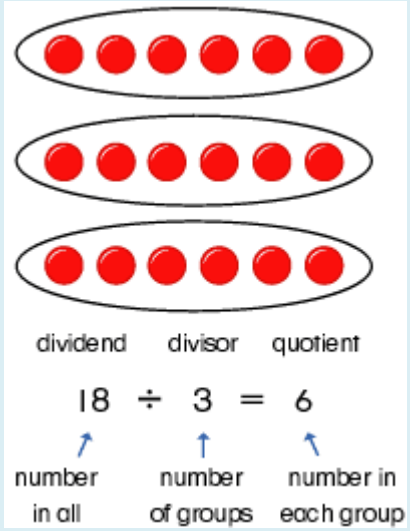
Objective and strategies	Concrete	Pictorial	Abstract
Counting in multiples ___multiples of ___ are equal to _____ 2, __, 6, 8, __, __	Count in multiples supported by concrete objects in equal groups.    Use concrete bar models to support.	Make representations to show counting in multiples.    	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 3, 6, 9, 12, 15 5, 10, 15, 20, 25, 30 Rolling numbers to count in multiples.  

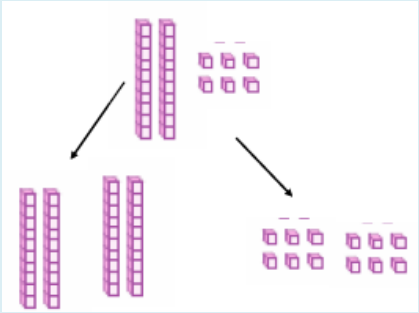
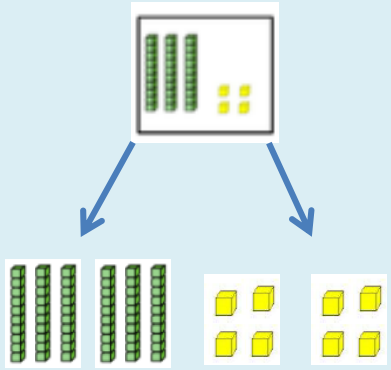
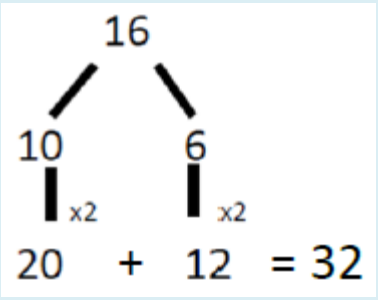
Objective and strategies	Concrete	Pictorial	Abstract
Scaling If ___ is increased by a scale factor ___ it is equal to _____	Use concrete resources such as objects, and increase using a scale factor. Make a tower 5 times as tall. 	Draw bar models and pictures to show scale factor. Increase by a scale factor of 5.  	Apply the knowledge of the relationship between scaling and multiplication to solve increasingly complex problems.  

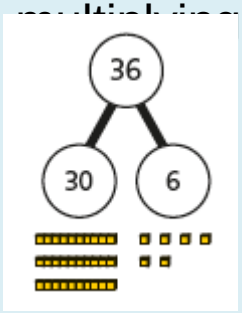
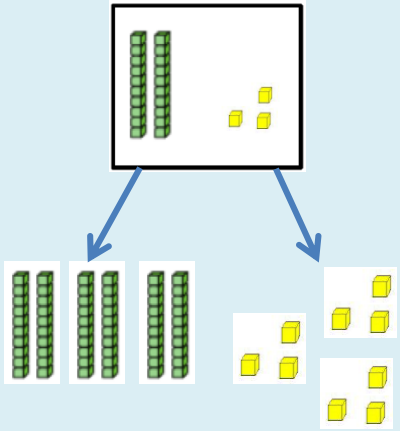
Objective and strategies	Concrete	Pictorial	Abstract
Correspondence _____ corresponds to _____ ___ x ___ = ____	Introduce the concept of correspondence problems where a number of objects are linked to a number of different objects using real life problems and concrete resources.  1 snowman, 4 hats, 2 scarves. How many combinations?	Draw the solution. Use a ruler and pencil to connect pictures so that they can count the connections. Begin recording the different combinations in a table or similar in a systematic order. Hat / Scarf Hat A / Scarf 1 Hat A / Scarf 2 Hat B / Scarf 1 Hat B / Scarf 2 Hat C / Scarf 1 Hat C / Scarf 2 Hat D / Scarf 1 Hat D / Scarf 2	Recognise the corresponding relationship as multiplication. There are 4 possible hats. For each hat there could be 2 possible scarves. The total number of ways to dress the snowman is $4 \times 2 = 8$ ways.

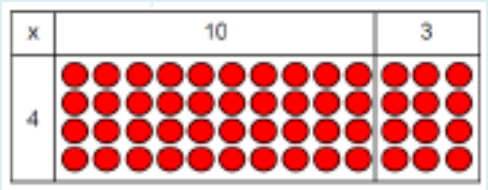
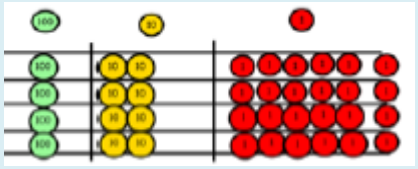
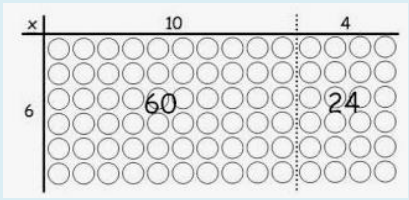
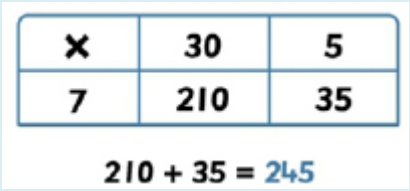
Objective and strategies	Concrete	Pictorial	Abstract
Understanding arrays – showing that multiplication is commutative factor x factor = product When you change the order of the factors, the **product** stays the same	Use objects laid out in arrays to find the answers to 2 groups of 5, 3 groups of 2 etc.   	Draw arrays in different rotations to find **commutative** multiplication s   4 x 3 = 3 x 4 	Use an array to write multiplication sentences and reinforce repeated addition.  3 + 3 + 3 + 3 = 12 4 x 3 = 12 

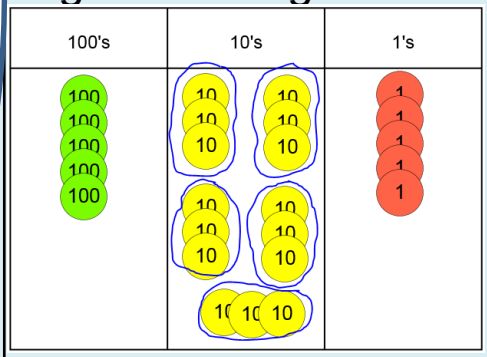
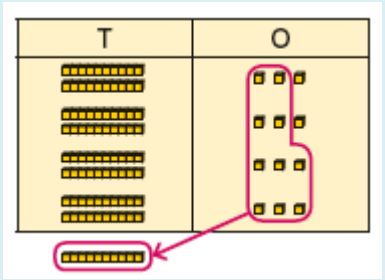
Objective and strategies	Concrete	Pictorial	Abstract
Multiplying 3 single digits __ x __ x __ = __ I multiply the product of two factors with the remaining factor.	Use objects, base 10, counters etc. Use commutative rules to multiply the product of two dice with the third dice.  1 x 5 x 3 = 15 	Use pictures or diagrams to solve. 2 x 6 x 7 = 2 x 42 = 84	Use commutative rules and/or brackets to solve. 2 x (8 x 5) = 2 x 40 = 80 Understand that if one of the factors is 0, the product will be 0.

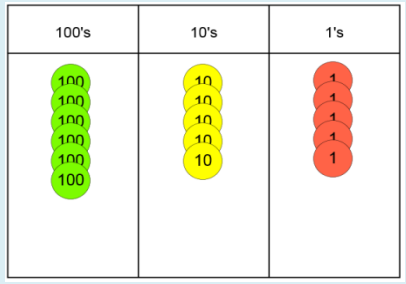
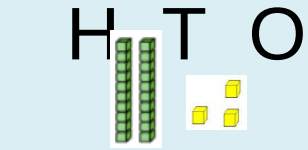
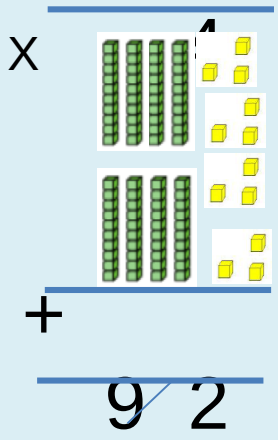
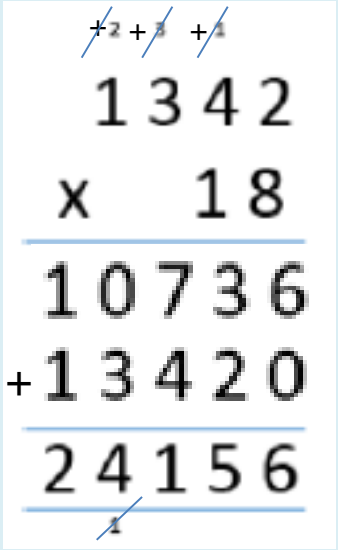
Objective and strategies	Concrete	Pictorial	Abstract
Using the inverse If I know that factor x factor = product Then I also know product ÷ factor = factor or dividend ÷ divisor = quotient *	Use objects to show inverse 5 groups of 4 cookies = 20 cookies altogether  20 cookies into groups of 4 cookies = 5 groups	Draw representations to show inverse.   	Write a fact family showing the link between multiplication and division. 3 x 4 =12 4 x 3 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3 

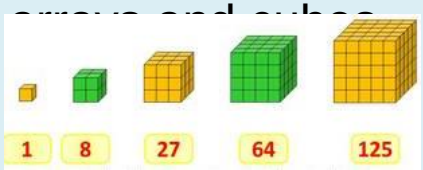
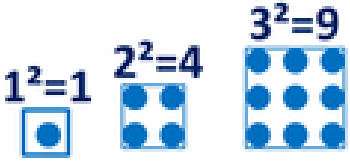
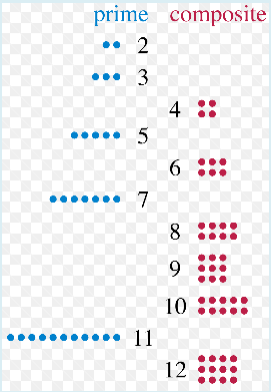
Objective and strategies	Concrete	Pictorial	Abstract
Doubling numbers beyond 16 = __ tens and __ ones. Double 10 = ___ Double 6 = ___ ___ + ___ = ___	Model doubling using base 10 equipment and place value counters.  Double 26 Double 20 = 40 Double 6 = 12 40 + 12 = 52	Draw pictures and representations to partition numbers before doubling. . Double 34  Double 30 Double 4 60 + 8 = 68	Partition a number and then double each part before recombining it back together. 

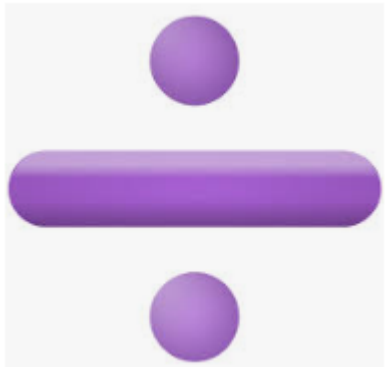
Objective and strategies	Concrete	Pictorial	Abstract
Partitioning to multiply ___ is __ tens and __ ones. Multiply the ones. Multiply the tens. __ tens + __ ones = ___	2 digit x 1 digit Use base 10, place value counters etc to partition two digit numbers before multiplying.  36 x 3 Children to solve practically.	Draw part-whole representations to partition numbers before multiplying. 23 x 3  60 + 9 = 69	Record as a number sentence using brackets, partitioning mentally where possible. 36 x 3 = (30 x 3) + (6 x 3) = 90 + 18 = 108

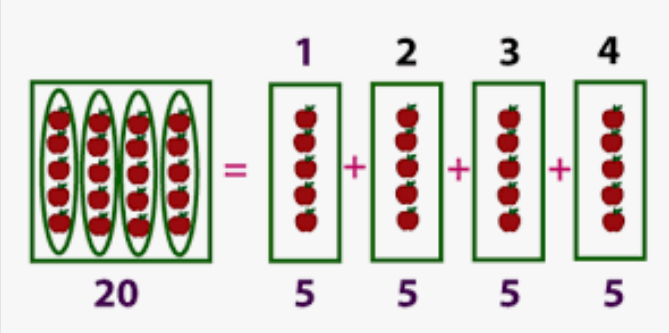
Objective and strategies	Concrete	Pictorial	Abstract
Grid method Step 1: partition the numbers into a grid. Step 2: multiply each box Step 3: Add the product of the boxes	Show the links with arrays to first introduce the grid method. 4 x 13  4 rows of 10, 4 rows of 3 Move on to base 10, place value counters etc:  400 + 80 + 24 = 504	Draw representations of place value counters or base 10 using a grid to organise the partitioned numbers. 14 x 6  60 + 24 = 84	Start with multiplying by one digit numbers, showing the clear addition alongside the grid.  Progress to multiplying 2 digit x 2 digit as appropriate.

Objective and strategies	Concrete	Pictorial	Abstract
Column multiplication. (Expanded) Step 1: multiply each digit by the ones Step 2: multiply each digit by the tens Step 3: add to find the product	Children can continue to be supported by place value counters at this stage of multiplication. Ensure resources are placed in columns and multiplied beginning with the least significant digit. 	Children can draw representations, multiplication rings etc for unknown tables, base 10 and/or place value counters in columns.  T O 2 3 x 4 1 2 (4 x 3) + 8 0 (4 x 20) 9 2	Formal expanded column multiplication, lining up digits clearly in columns. Use brackets to record each calculation to TO 32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768

Objective and strategies	Concrete	Pictorial	Abstract
Column multiplication (concise) Step 1: multiply each digit by the ones (exchange if necessary) Step 2: Insert place holder and multiply each digit by the tens (exchange if necessary) Step 3: add to find the product (exchange if necessary)	Base 10, place value counters should be used to  131 x 5 = 500 + 150 + 5 Regroup 150 as 1 hundred and 5 tens.  	Draw representations, multiplication rings for unknown tables, base 10 and/or place value counters in columns.   	Regrouping should be shown above the calculation with a + symbol. Addition regrouping is shown below the 

Objective and strategies	Concrete	Pictorial	Abstract
Square, cube, prime composite numb A prime number is___ A composite number is ___ A square number is ___ A cube number is___	Build representations of square, cube, prime and composite numbers using arrays and cubes  	Draw squares, cubes and rectangles to represent square, cube and prime numbers.  	Recognise and record prime numbers. Use notation to record square and cube numbers. $1^2 = 1 \times 1 = 1$ $2^2 = 2 \times 2 = 4$ $3^2 = 3 \times 3 = 9$ $4^2 = 4 \times 4 = 16$ $5^2 = 5 \times 5 = 25$ $6^2 = 6 \times 6 = 36$ $7^2 = 7 \times 7 = 49$ $8^2 = 8 \times 8 = 64$ $9^2 = 9 \times 9 = 81$ $10^2 = 10 \times 10 = 100$



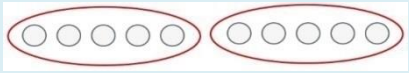
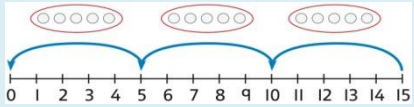


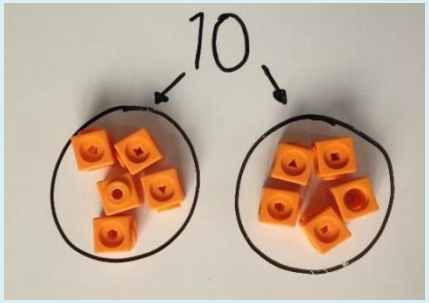
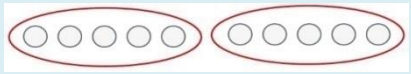
Division

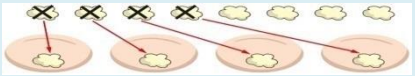
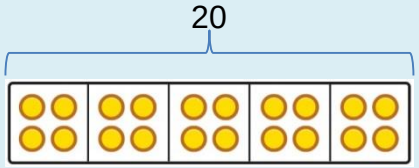
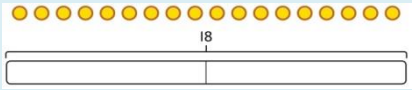
Key Vocabulary:

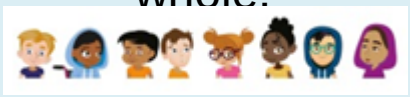
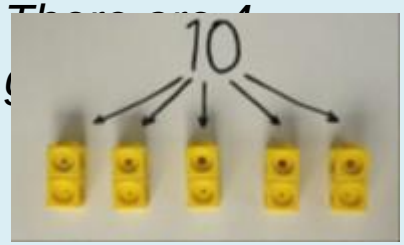
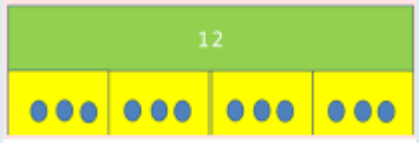
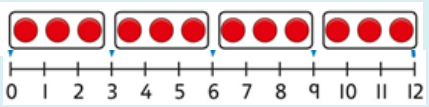
share, equal, total, divide, group, short division, long division, halve, partition, place value, remainder, multiply, divide, inverse, fact family, add, subtract, factor, multiple, exchange

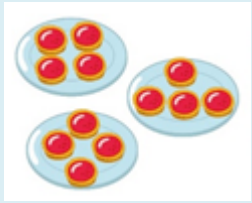
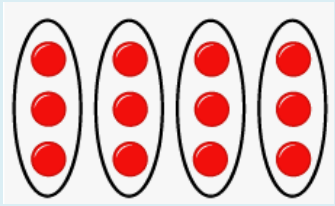
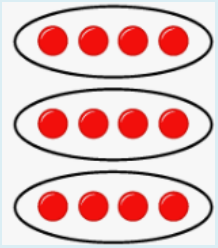
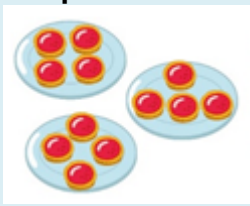
Teachers need to model the language of dividend ÷ divisor = quotient (remainder)

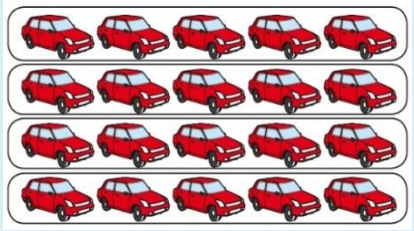
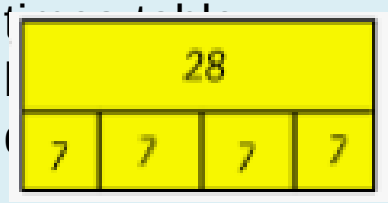
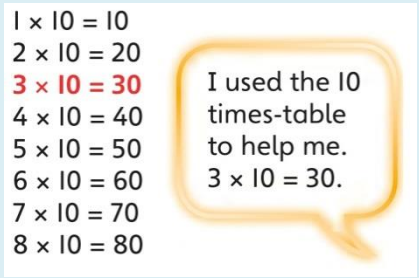
Objective and strategies	Concrete	Pictorial	Abstract
Making equal groups There are __in each group. There are __ groups	Sort a whole set people and objects into equal groups.  *There are 10 children altogether.* *There are 2 in each group.* *There are 5 groups.*	Represent a whole and work out how many equal groups.  *There are 10 in total.* *There are 5 in each group.* *There are 2 groups.*	Children may relate this to counting up in steps of 2, 5 or 10. 

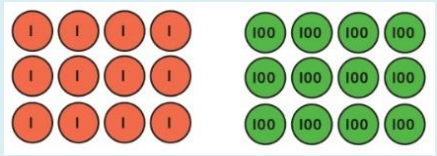
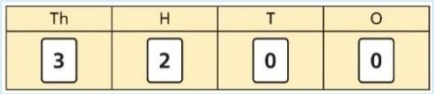
Objective and strategies	Concrete	Pictorial	Abstract
Halving Half of __ is equal to ___	Share a whole set people and objects between two.  *There are 10 sweets altogether. They are shared between 2 people. There are 5 sweets each.*	Use pictures as representations to halve even numbers up to 10, progressing to even numbers up to 20   *There are 10 sweets altogether. They are shared between 2 people. There are 5 sweets each.*	Half of 6 is 3. 3 is half of 6. If I halve 6 I get 3. Children to make a link to 2 times tables, and to doubling as inverse facts.

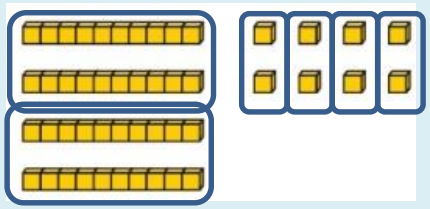
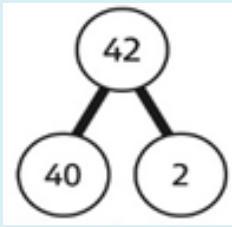
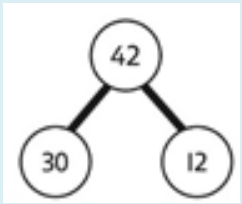
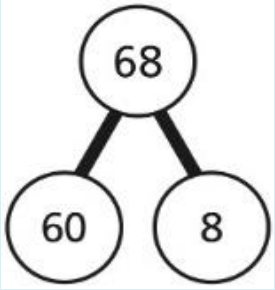
Objective and strategies	Concrete	Pictorial	Abstract
Sharing equally __ shared into __ equal parts. There are __ in each part.	Start with a whole and share into equal parts, one at a time.  8 shared equally between 4 plates. There are 2 on each plate.	Sketch or draw to represent sharing into equal parts.  Represent the objects shared into equal parts using a bar model.  *20 shared into 5 equal parts. There are 4 in each part.*	*10 shared into 2 equal parts gives 5 in each part.* Use a bar model to support understanding of the division.  *18 ÷ 2 = 9* *Introduce language of dividend, divisor and quotient.*

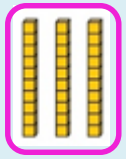
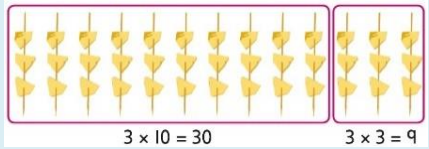
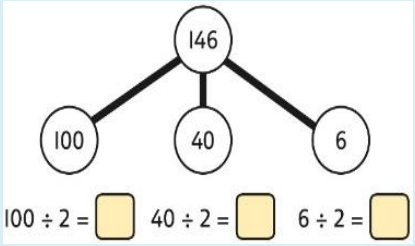
Objective and strategies	Concrete	Pictorial	Abstract
Grouping __ divided into groups of __. There are __ groups.	Understand how to make equal groups from a whole.   *8 divided into groups of 2.* *There are 4*  *10 divided into groups of 2.* *There are 5*	Understand the relationship between grouping and the division statements. 12 ÷ 6 = 2  12 ÷ 4 = 3  12 ÷ 2 = 6  12 ÷ 3 = 4  Draw bar models to represent grouping  	Understand how to relate division by grouping.  *12 divided into groups of 3.* *12 ÷ 3 = 4* *There are 4 groups.*

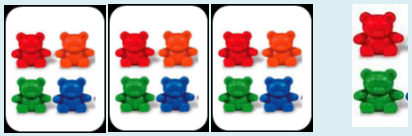
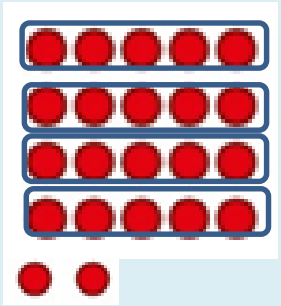
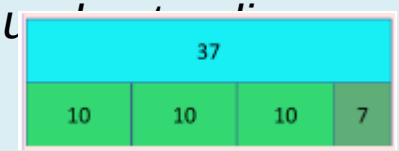
Objective and strategies	Concrete	Pictorial	Abstract
Link sharing and grouping __ shared into __ equal parts is __. __ into groups of __ is __.	Use objects to practically explore the language of sharing and grouping.  Share 12 cakes between 3 plates. Group 12 cakes into groups of 4.	Use pictures to represent grouping and sharing. What is the same? What is different? 12 ÷ 3 = 4 Grouping:  Sharing: 	Describe a picture using the language of sharing and grouping. One picture, two  12 cakes shared between 3 plates. There are 4 on each plate. 12 ÷ 3 = 4 12 cakes into groups of 4.There are 3 groups. 12 ÷ 4 = 3

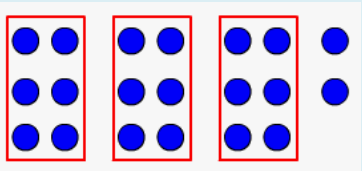
Objective and strategies	Concrete	Pictorial	Abstract
Using known times-tables to solve divisions If I know that factor x factor = product Then I also know product ÷ factor = factor or dividend ÷ divisor = quotient	Understand the relationship between multiplication facts and division.  *4 groups of 5 cars is 20 cars in total.* *20 divided into groups of 5 is 4* *20 ÷ 5 = 4*	Link equal grouping with rolling numbers and known times-table facts to support division  *12 divided by 4 is 3.* Use a bar model to support understanding of the link between 	Relate times-table knowledge directly to division  *I know that 3 groups of 10 makes 30, so I know that 30 divided into groups of 10 is 3.* *3 × 10 = 30 so 30 ÷ 10 = 3*

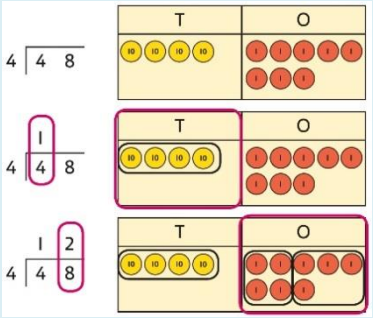
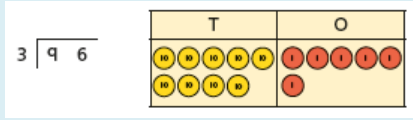
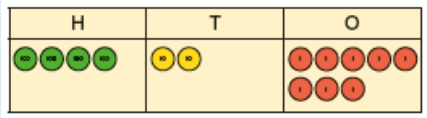
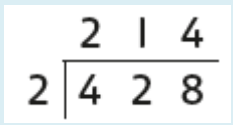
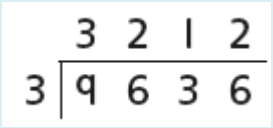
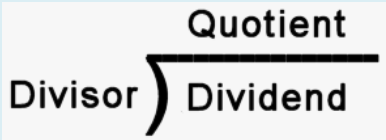
Objective and strategies	Concrete	Pictorial	Abstract
Dividing whole numbers by 10, 100 and 1,000 To divide by 10, the digits move __ spaces to the right. To divide by 100, the digits move __ spaces to the right. To divide by 1000, the digits move __ spaces to the right.	Use place value equipment to support unitising for division.  *15 ones put into groups of 3 ones. There are 5 groups.*  *15 tens put into groups of 3 tens. There are 5 groups.* *150 ÷ 30 = 5*	Represent related facts with place value equipment when dividing by unitising.  *12 ones divided into groups of 4. There are 3 groups.* *12 hundreds divided into groups of 4 hundreds. There are 3 groups.* *1200 ÷ 400 = 3*	Understand how and why the digits change on a place value grid when dividing by 10, 100 or 1,000.  *3,200 ÷ 100 = ?* *200 ÷ 100 = 2* *3,000 ÷ 100 = 30* *3,200 ÷ 100 = 32* *So, the digits will move two places smaller (to the right).*

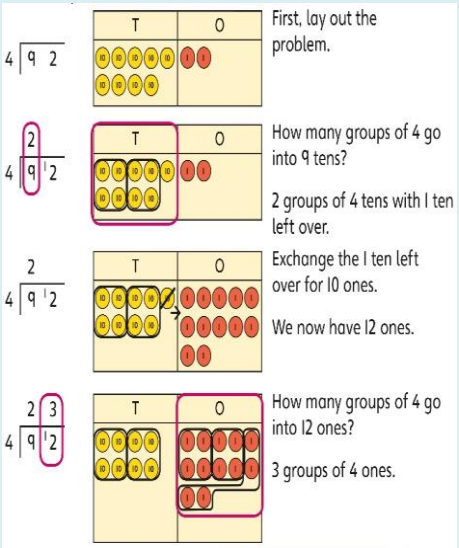
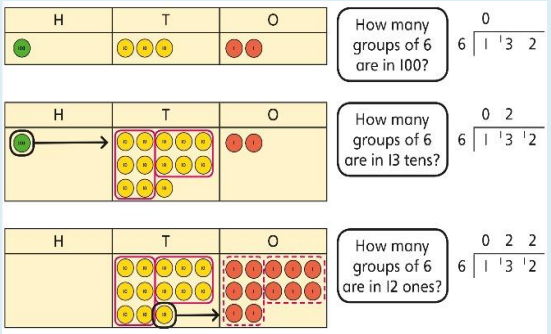
Objective and strategies	Concrete	Pictorial	Abstract
2-digit number divided by 1-digit number, no remainders (flexible partitioning) Partition the dividend into tens and ones. ___ is __ tens and __ ones. Divide the tens. __ ÷ __ = __ Divide the ones. __ ÷ __ + __ __ tens + __ ones = ___	Children explore dividing 2-digit numbers by using place value equipment. *48 ÷ 2 = ?*  2 groups of tens and 4 groups of ones. *48 ÷ 2 = 24*	Children explore which partitions support particular divisions.  42 ÷ 2  42 ÷ 3	Children partition a number into 10s and 1s to divide where appropriate.  *60 ÷ 2 = 30* *8 ÷ 2 = 4* *30 + 4 = 34* *68 ÷ 2 = 34* Use flexible partitioning using known facts.

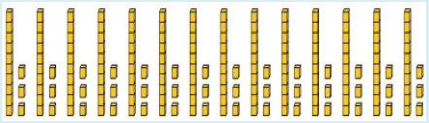
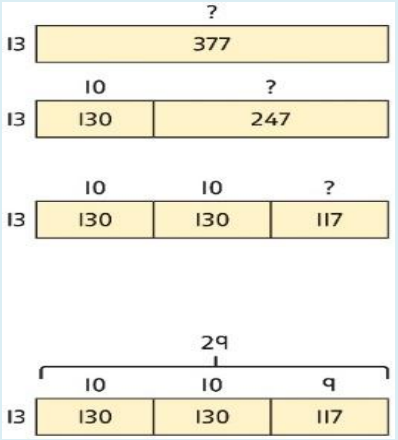
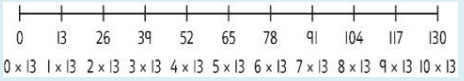
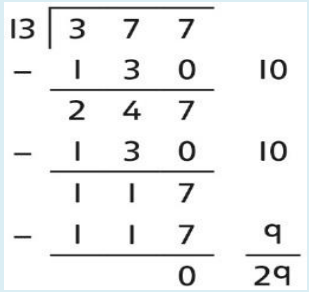
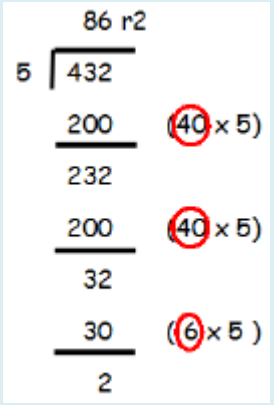
Objective and strategies	Concrete	Pictorial	Abstract
Dividing up to 3-digit numbers by a single digit by partitioning into 100s, 10s and 1s See previous slide – extend to 3 digit numbers	Partition into 10s and 1s to divide where appropriate. *39 ÷ 3 = ?*   One group of tens 3 groups of ones *39 = 30 + 9* *30 ÷ 3 = 10* *9 ÷ 3 = 3* *39 ÷ 3 = 13*	Partition into 100s, 10s and 1s using jottings to divide where appropriate. *39 ÷ 3 = ?*  *39 = 30 + 9* *30 ÷ 3 = 10* *9 ÷ 3 = 3* *39 ÷ 3 = 13*	Partition into 100s, 10s and 1s using a part-whole model to divide where appropriate. *142 ÷ 2 = ?*  *100 ÷ 2 = 50* *40 ÷ 2 = 20* *6 ÷ 2 = 3* *50 + 20 + 3 = 73* 142 ÷ 2 = 73

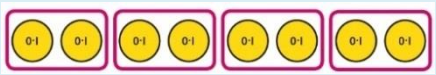
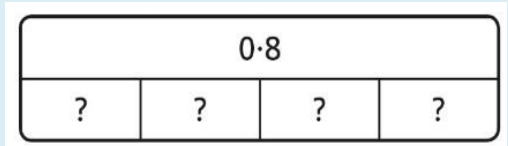
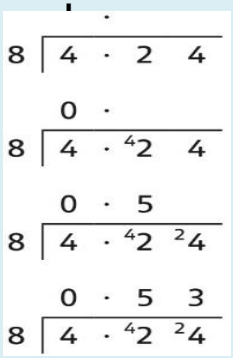
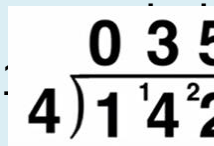
Objective and strategies	Concrete	Pictorial	Abstract
Understanding remainders There are __ equal groups and __ left over (remaining) . Dividend ÷ divisor = quotient and remainder	Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further.  *There are 13 sticks in total. There are 3 groups of 4, with 1 remainder.*  *14 ÷ 4 = 3 with 2 remaining.*	Use images to explain remainders.  *22 into groups of 5 is 4 groups and 2 remainders* *22 ÷ 5 = 4 r2* *Use a bar model to support* 	Understand that the remainder is what cannot be put into an equal group. *22 ÷ 5 = ?* *5* *10* *15* ~~*20*~~ *25 (25 is more than 22)* *There are 4 equal groups of 5.* *Count on from 20 to 22.* *There are 2 remainders* *22 ÷ 5 = 4 r2* 

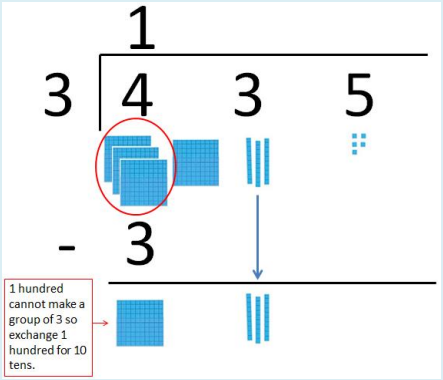
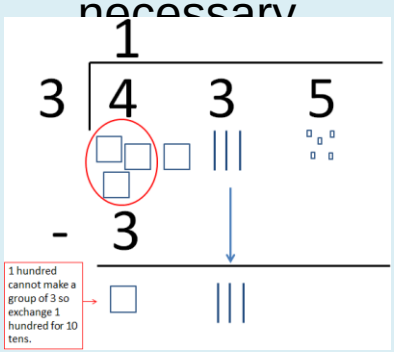
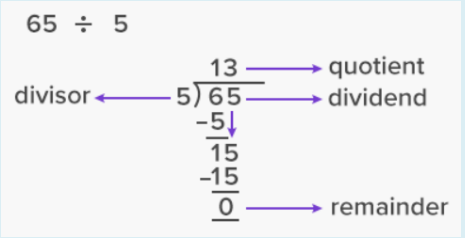
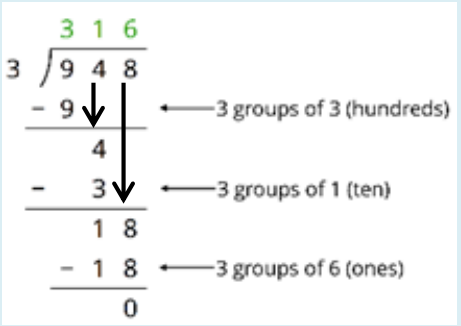
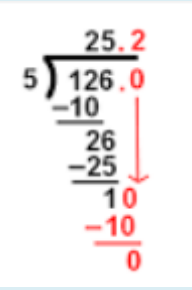
Objective and strategies	Concrete	Pictorial	Abstract
2-digit number divided by 1-digit number, with remainders Step 1: partition the dividend into tens and ones. __ = __ tens and __ ones Step 2: divide the tens __ ÷ __ = __ Step 3: divide the ones __ ÷ __ Write the remainder.	Use place value equipment to understand the concept of remainder. *Make 29 from place value equipment.* *Divide into groups of 2*  *1 group of tens and 4 groups of ones and 1 remainder.* *29 ÷ 2 = 14 r1*	Use drawings to understand the concept of remainder in division. *20 ÷ 6 = ?*  *20 ÷ 6 = 3 remainder 2*	Partition to divide, understanding the remainder in context. *67 children try to make 5 equal lines.* *67 = 60 + 7* *60 ÷ 5 = 12* *7 ÷ 5 = 1 remainder 2* *67 ÷ 5 = 13 r2* *There are 13 children in each line and 2 children left out.*

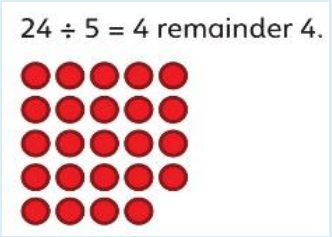
Objective and strategies	Concrete	Pictorial	Abstract
Dividing up to four digits by a single digit using short division (no exchange) Set out the short division. Divide the tens. __ ÷ __ Write the quotient in the tens column Divide the ones. __ ÷ __ Write the quotient in the ones column*	Use place value equipment on a place value grid alongside short division. *48 ÷ 4 = ?*  *There is 1 group of 4 tens.* *There are 2 groups of 4 ones.* *48 ÷ 4 = 12*	Use jottings to show understanding of place value within short division. 96 ÷ 3 =    	Progress to up to 4 digits divided by a single digit as appropriate using short division.  Link layout of formal method with language of number sentence. 

Objective and strategies	Concrete	Pictorial	Abstract
Dividing up to four digits by a single digit using short division (with exchange) Set out the short division. Divide the tens. __ ÷ __ (exchange where necessary) Write the quotient in the tens column Divide the ones. __ ÷ __ (exchange where necessary) Write the quotient in the ones column*	Use base 10/place value equipment on a place value grid alongside short division. *Ensure exchange is explored practically and verbal explanations to show understanding.* 	Use jotting alongside short division. Work with divisions that require exchange.  **How many groups of 6 are in 1 hundred.* This needs explicit teaching to ensure children understand that they are making groups of 6 using hundreds. Use base 10 to reinforce place value understanding.	Use short division for up to 4-digit numbers divided by a single digit. *3,892 ÷ 7 = 556* 0 5 5 6 7 | 3 ³8 ³9 ⁴2 Use multiplication to check. *556 × 7 = ?* *6 × 7 = 42* *50 × 7 = 350* *500 × 7 = 3500* *3,500 + 350 + 42 = 3,892* Divisions with a remainder explored in problem-solving contexts.

Objective and strategies	Concrete	Pictorial	Abstract
Dividing by a 2-digit number using long division with multiples Step 1: subtract known multiples of the divisor. Step 2: subtract the remaining multiples until you have reduced all of the dividend. Step 3: add the total of the multiples. Write the quotient and any remainders	Use equipment to build numbers from groups.  *182 divided into groups of 13. There are 14 groups.*	Use an area model alongside written division to model the process.  *377 ÷ 13 = 29*	Use long division. Write the required multiples to support the division process.   *377 ÷ 13 = 29* 

Objective and strategies	Concrete	Pictorial	Abstract
Dividing decimals Set out the short division. Divide the ones. __ ÷ __ (exchange where necessary) Write the remainder in the tenths column Write the decimal point. Divide the tenths. __ ÷ __ (exchange where necessary) Write the remainder in the hundredths column*	Use place value equipment to explore division of decimals.  *8 tenths divided into 4 groups. There are 2 tenths in each group.* *0.8 ÷ 4 = 0.2*	Use a bar model to represent divisions.  4 × 2 = 8 8 ÷ 4 = 2 So, 4 × 0·2 = 0·8 0·8 ÷ 4 = 0·2	Use short division to divide decimals with up to 2 decimal  Extend to decimal  142 ÷ 4 = 35·5 2/4 =1/2= 0.5

Objective and strategies	Concrete	Pictorial	Abstract
Long division (formal method) The stem is the script in the notes section (in bold) . This must be used consistently to ensure understandin g	Use base 10 equipment and a bus stop template to divide practically. Encourage children to record their workings on the template as they group and subtract.  Continue with exchange and regrouping to make groups of 3	Use pictorial representations of long division using base 10 or equivalent. Children can circle groups and record alongside concrete resources if necessary.  	Use long division. Divide by a 2 digit number including a decimal dividend.   Extend to decimals. 

Objective and strategies	Concrete	Pictorial	Abstract
Understanding factors and prime numbers A prime number is___ A composite number is __ A factor is __	Use equipment to explore the factors of a given number.  *24 ÷ 3 = 8* *24 ÷ 8 = 3* *8 and 3 are factors of 24 because they divide 24 exactly.*  5 is not a factor of 24 because there is a remainder.	Understand that prime numbers are numbers with exactly two factors. *13 ÷ 1 = 13* *13 ÷ 2 = 6 r 1* *13 ÷ 3 = 4 r 1* *13 ÷ 4 = 3 r 1* *13 ÷ 5 = 2 r 3* *13 ÷ 6 = 2 r 1* *1 and 13 are the only factors of 13. 13 is a prime number.*	Understand how to recognise prime and composite numbers. *I know that 31 is a prime number because it can be divided by only 1 and itself without leaving a remainder.* *I know that 33 is a composite number (not a prime number) as it can be divided by 1, 3, 11 and 33.* *I know that 1 is not a prime number, as it has only 1 factor.*